Here is a Matlab script that demonstrate multi-way spectral graph clustering. It is recommended that you run the script step by step (cut and paste), relating each step to the class slides. As you go along, answer the questions posed in red.

```matlab
%% Load graph data
load AvgCorrelHC.mat;
load annotatedStructTemplate90codeBook.mat;

%% Graph matrices
% adjacency matrix
A=C;

% setup degree vector
D=sum(A);

% graph Laplacian
L=diag(D)-A;

% normalized adjacency matrix
An=diag(1./sqrt(D))*A*diag(1./sqrt(D));

% normalized graph Laplacian
Ln=eye(length(An))-An;

%% Graph spectrum
[eV,eD]=eig(Ln);
figure(1);
plot(diag(eD));
xlabel('index');
ylabel('eigenvalue');

%% Get multiple clusters
% number of eigenvectors to keep for clustering
CONST_K=10;
V=eV(:,2:CONST_K+1);

% normalize (according to Ng, Jordan, Weiss 2002)
V=V./repmat(sqrt(sum(V.^2,2)),[1 CONST_K]);

CONST_CLUSTERS=7; % number of cluster
CONST_MRKS={'ro','bo','go','kp','co','mo','yo'};
[idx,centroids]=kmeans(V,CONST_CLUSTERS,'replicates',20);

figure(2);
for iter=1:CONST_CLUSTERS,
    fprintf('Cluster %d:
',iter);
    tmp=find(idx==iter);
    codeBookStructTemplate.full.name(tmp)
    plot3(xyz(1,tmp),xyz(2,tmp),xyz(3,tmp),CONST_MRKS{iter},'MarkerSize',9);
    tmplegend(iter)=sprintf('cluster %d',iter);
    hold on;
end;
```

Visualize the different graph properties: adjacency matrix (using `imagesc`), degree vector (using `plot`), graph Laplacian and its normalized version (using `imagesc`).

Take a look at the first three eigenvectors. Can you qualitatively interpret them like low-frequency components on the graph?

Indeed, the first one plays the role of the “constant”. The following ones seem to increase their “frequency” on the nodes of the graph.

Multi-way clustering takes multiple eigenvectors into a k-means clustering algorithm. Is the solution unique? Try varying the #retained eigenvectors and #clusters.

No. The clustering starts with a random initialization each time.
hold off;
grid on;
axis equal;
legend(tmplegend);
xlabel('x');
ylabel('y');
zlabel('z');