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# Graph Signal Processing: Foundations and Emerging Directions

urs is a connected world where unprecedented quantities of data are being generated, recorded, and processed [1]. Generically, the networks that sustain our societies can be understood as complex systems formed by multiple nodes, where global network behavior arises from local interactions between connected nodes. More succinctly, a network or a graph can be defined as a structure that encodes relationships between pairs of elements of a set. The simplicity of this definition drives the application of graphs and networks to a wide variety of disciplines, such as biology, medicine, psychology, sociology, economics, engineering, computer science, and so on [2]. Often, networks have intrinsic value and are themselves the object of study. On other occasions, the network defines an underlying notion of proximity, but the object of interest is the information defined on top of the graph, i.e., data associated with the nodes of the network.

Graph signal processing (GSP) has approached this problem by modeling the structure of the data using a graph and then viewing the available information as a signal defined on it. A plethora of graph-supported signals exists in different engineering and scientific fields, with examples ranging from gene-expression patterns defined on gene networks; the spread of epidemics, rumors, or memes over a population; or the congestion level at the nodes of a telecommunication net-

Digital Object Identifier 10.1109/MSP.2020.3020715 Date of current version: 28 October 2020 work, to name a few. More generally, GSP arises in a broad set of application domains, from social networks to the business and corporate worlds and from health care to critical cyberphysical infrastructures.

Transversal to the particular application, the SP community can contribute to the advancement of the understanding of network data by redesigning traditional tools originally conceived to study signals defined on regular domains (such as time-varying signals) and extend them to analyze signals on the more complex graph domain. This leads to the development of new signal models, data analytics, and algorithms that, by incorporating

the underlying network structure, provide the opportunity for gains in the accuracy and efficiency of processing the relevant information present in the network data. In this context, the theoretical and practical success achieved by GSP in the last few

years has been noticeable and includes, for example, the generalization of tools such as frequency analysis, filtering, sampling, or statistical stationarity to signals supported on graphs [3], along with their practical use in applications as important as video processing and neuroscience [4], [5]. Nonetheless, many issues remain open, with, e.g., robust, nonlinear, or higher-dimensional GSP being at their infancy. This status is, of course, not surprising. Human knowledge about time-varying signals and images was developed over the course of decades and boosted by real needs in areas such as communications, speech, video, or control. In contrast, the prevalence of network-related SP problems and access to quality network data are recent phenomena. However, the explosion of available network data is generating a pressing need to better understand and process information in network settings that is expected to not only foster the development of GSP but to also cement its relevance via its application to a growing number of problems.

Our goal as guest editors is for the SI to not only introduce SP researchers to the field of GSP and to some of its most recent advances but to also broaden the impact of the field by building bridges to related area. This is the setting that motivates the publication of this special issue (SI) of *IEEE Signal Processing Magazine (SPM)*. Our goal as guest editors is for the SI to not only introduce SP researchers to the field of GSP and to some of its most

recent advances but to also broaden the impact of the field by building bridges to related areas, such as machine learning, statistics, and data science as well as to identify additional relevant applications especially suited to being addressed using GSP tools. The ultimate objective is to serve as a catalyst to accelerate the generation of GSP-related results, hence contributing to the advancement and understanding of the field.

## An overview of the SI

To achieve the aforementioned goals, the SI starts with four articles overviewing current trends in classical GSP tasks, including sampling, spectral frames, and filtering. In particular, Tanaka et al. discuss the similarities and differences between sampling standard signals and signals defined over graphs, review current progress on sampling over graphs, and highlight open problems focusing on theory and potential applications. Also, in the context of sampling, the article by Lau et al. presents a family of methods developed under the umbrella of blue-noise sampling on graphs, a simple and intuitive principle that generates patterns with low computational cost without requiring a spectral decomposition, providing an alternative to those existing methods that require the calculation of eigenvalues and eigenvectors. Shuman looks at the problem of how to build dictionaries of atoms to efficiently represent signals defined over a graph. The main interest of his article is in a class of dictionaries called *localized spectral* 

graph filter frames, which encompasses a variety of approaches from spectral graph wavelets to graph filter banks, analyzing in detail the design and application of such frames. Finally, Teke and Vaidyanathan revisit the

definition of a linear filter and analyze how it can be leveraged in the context of asynchronous and distributed network implementations. Their article not only reviews recent results on this topic but also highlights the relevance of GSP in the context of analyzing and designing distributed network operators. Building on many of the concepts and tools reviewed in these manuscripts, Ramakrishna, Wai, and Scaglione demonstrate how low-pass graph signals and filters are prevalent in science and engineering. They then leverage this structure to develop tailored designs and algorithms that enhance recovery performance.

The next two articles aim to broaden the applicability of GSP results. The first, by Dittrich and Matz, looks at setups where the supporting graphs are unweighted but signed, so that the links of the graphs can be used to naturally account for similarity and dissimilarity pairwise relationships. The article revisits many of the classical GSP results (from sampling to topology inference) and presents new algorithms for setups where the edges of the graph are signed. Marques, Segarra, and Mateos provide an overview of GSP over directed graphs, identifying the current GSP tools and results that (with minor modifications) can be applied to digraphs as well as the main opportunities for and challenges to developing a comprehensive framework of GSP over digraphs.

The SI then shifts gears to three articles looking at the connections between GSP and machine learning. The article by Dong et al. reviews how GSP has contributed to the development of novel machine learning algorithms, where the analysis and representation (includ-

The ultimate objective is to serve as a catalyst to accelerate the generation of GSP-related results, hence contributing to the advancement and understanding of the field. ing visualization) of large-scale structured data are of paramount importance. The article emphasizes how GSP is especially well suited for exploiting data structure and relational priors, improving data and computational effi-

ciency, and enhancing model interpretability. Gama et al. look at the relationship between graph filters and graph convolutional neural networks. Their article starts with linear graph filters as natural graph-signal operators, analyzes their strengths and weaknesses, and then presents graph convolutional neural networks as a generalization capable of overcoming many of the limitations present in linear graph filters. The triadic closes with the article by Cheung et al., which reviews in detail how graph filters can be used in graph-aware deep neural network architectures and then exploits those for learning applications dealing with graph signals.

The last part of the SI is devoted to a promising emerging GSP direction: developing SP tools that go beyond pairwise or local interactions. The first article, written by Petrović et al., looks at modularity-derived graph spectral domain to define communityaware GSP operations. After highlighting how essential it is to leverage the multiway relationships present in most modern data sets, Stanley, Chi, and Mishne discuss modern SP frameworks generalizing GSP to multiway data, synthesizes common themes arising from current efforts to combine GSP with tensor analysis, and highlights future directions in extending GSP to the multiway paradigm. Finally, the article by Barbarossa and Sardellitti also looks at multiway relationships, but in this case, from the point of view of simplicial complexes. The article advocates for the advancement of a theory of topological SP, overviews existing results, and illustrates how GSP can be viewed as a prominent special case of that approach. Although certainly relevant, the topic of using GSP tools to learn a graph from data has purposely been left out from the SI because SPM has recently published two comprehensive feature articles on this topic [6], [7].

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We would like to express our gratitude and appreciation to a number of people. First, a thank you to the authors of the more than 50 white papers submitted to the SI. Unfortunately, we could only accept some of them due to space constraints. We do regret that so many good papers did not make it to the final selection. Second, a thank vou to the editorial board and staff of SPM and, especially, to Senior Editorial Board Member David Love, for encouraging, reviewing, welcoming, and facilitating the processing of articles to be included. Last but not least, thank you to the conscientious reviewers for their volunteer efforts, constructive feedback, and timely responses. This SI would not have been possible without all of their dedicated work.

#### **Guest Editors**



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